On the Zitterbewegung and internal dynamics of the electron

Olivier Rousselle

Fondation Louis de Broglie, 23 rue Marsoulan, 75012 Paris, France
Sorbonne Université, 4 place Jussieu, 75005 Paris, France

RÉSUMÉ. Cet article se concentre sur la compréhension de l'électron et de ses propriétés intrinsèques. Nous analysons le mouvement de Zitterbewegung, qui renseigne sur la dynamique interne de l'électron (avec un mouvement interne à la vitesse de la lumière et une amplitude $\sim 10^{-13}$ m). Ce mouvement de zitterbewegung est directement lié au moment angulaire de spin. Cet article s'inscrit dans la continuité des idées de De Broglie et d'Einstein visant à rendre la physique quantique réaliste (avec des trajectoires réelles et des ondes physiques) et à définir une structure géométrique à l'électron.

ABSTRACT. This paper focuses on the understanding of the electron and its intrinsic properties. We analyze the Zitterbewegung motion, that gives information about the internal dynamics of the electron (with internal motion at the speed of light on average and an amplitude $\sim 10^{-13}$ m). This zitterbewegung motion is directly connected to the spin angular momentum. This article is in the continuity of De Broglie and Einstein ideas to make quantum physics realistic (with real trajectories and physical waves) and to define a geometrical structure to the electron.

Introduction

At the beginning of the 20^{th} century, physicists discovered that light has a double nature: wave (with frequency and wavelength) and particle (with energy and momentum). Those two faces of light are related by the formula $E = h\nu$. In his thesis of 1924 [1], Louis de Broglie conjectured that this duality particle-wave can be generalized to matter (like the electron). The wave nature of matter was confirmed in 1928 by

Davisson and Germer [2]. However, De Broglie described the particle as a singularity of the wave, which was not satisfactory; according to him, physics has an urgent need to define a structure of elementary particles and, in particular, to introduce an electron radius. Today, the electron is described in quantum mechanics as point-like, with the charge and spin considered as intrinsic and with infinite self-energy.
The first structural model of the electron was proposed by Parson [3], in which the electron has a ring-shaped geometry where the unitary charge moves around a ring at the speed of light generating a magnetic field. In 1930, Schrödinger showed from the Dirac equation that the electron has a high frequency oscillatory motion of small amplitude (on the order of Compton wavelength), superposed on the regular motion [4]; he called it "Zitterbewegung" (trembling movement in German). More recently, Huang [5] and Hestenes [6] interpreted the Zitterbewegung as an internal helical motion generating the properties of the electron (charge, spin and magnetic moment), and Williamson described the electron as a photon confined in toroidal topology [7]. Moreover, circulating solutions of Maxwell's equations have been found to exist [8].

In part 1 of the article, we will analyze the Zitterbewegung equations of motion coming from the Dirac equation. In part 2, we will study in detail the properties of the electron at rest. Finally, the part 3 focuses on the electron in motion and the link with special relativity. Note that this article focuses on the electron, but it applies for all elementary particles obeying Dirac equation (like muon, tau).

1 Zitterbewegung motion

1.1 Interpretations of Zitterbewegung

The main interpretation of the Zitterbewegung is that interference between positive and negative energy states produces an apparent fluctuation (up to the speed of light) of the position of an electron around the median, with an angular frequency of $2mc^2/\hbar$ [9]. However, the Zitterbewegung equations of motion can also be derived from the Levy-Leblond equation, which is the wave equation for non-relativistic particles of spin 1/2 [10]; this equation doesn't involve negative energies of course, which contradicts the interpretation with the interference between positive and negative energies. Moreover, the zitterbewegung effect cannot be due to vacuum fluctuations (virtual photons and electron-positron pairs) since the creation/annihilation operators don't appear in the Dirac equation.

Finally, we will see that the motion of the Dirac particle is at the speed of light in Zitterbewegung equations; this velocity cannot be a longitudinal velocity (since the electron is a massive particle), so it should be an internal velocity. In this article, we will follow the realistic interpretation of D. Hestenes, which consists of a physical internal motion of the electron (light-like helical motion at the speed of light); this motion gives rise to the spin and magnetic moment of the electron in a concrete geometric way.

1.2 Zitterbewegung equations

The Dirac Hamiltonian of a free particle is:

$$\hat{H} = \hat{\beta} mc^2 + c\sum_{i=1}^{3} \hat{\alpha}_i \hat{p}_i, \tag{1}$$

with m the mass of the particle, $\hat{p}_i$ the momentum operator, and $\hat{\beta}$, $\hat{\alpha}_i$ are related to the 4×4 Dirac matrices γ_μ ($\beta = \gamma_0$ and $\alpha_i = \gamma_0\gamma_i$, with $\alpha_i^2 = I$). The Hamiltonian operator $\hat{H}$ that describes the relativistic dynamics of spin-1/2 particles is then a matrix 4×4, acting on the bispinor $\psi(\boldsymbol{x}, t)$. The energy eigenvalue of the Dirac Hamiltonian for a free particle is $E = \sqrt{m^2c^4 + p^2c^2}$. The α matrices encapsulate the coupling of spin with particle motion; this is related to the spinor structure of the Dirac equation. We have the following commutation relations:

$$[\hat{x}_j, \hat{p}_k] = i\hbar\delta_{jk} \ , \ [\hat{x}_j, \hat{\alpha}_k] = 0 \ , \ [\hat{p}_j, \hat{\alpha}_k] = 0 \ , \ [\hat{H}, x_k] = -i\hbar c\hat{\alpha}_k. \tag{2}$$

In the Heisenberg picture, a time-dependent observable obeys the following equation:

$$\frac{d\hat{O}(t)}{dt} = \frac{i}{\hbar}\hat{U}^\dagger(t)[\hat{H}, \hat{O}]\hat{U}(t), \tag{3}$$

with $\hat{U}(t) = e^{-i\hat{H}t/\hbar}$ time evolution operator. The time evolution of the $\hat{\alpha}$ operator is then:

$$\frac{d\hat{\alpha}_k(t)}{dt} = \frac{i}{\hbar}\hat{U}^\dagger(t)[\hat{H}, \hat{\alpha}_k]\hat{U}(t) = \frac{2i}{\hbar}(c\hat{p}_k - \hat{\alpha}_k(t)\hat{H}). \tag{4}$$

By integrating this equation, we get:

$$\hat{\alpha}_k(t) = (\hat{\alpha}_k - c\hat{p}_k\hat{H}^{-1})e^{-\frac{2i\hat{H}t}{\hbar}} + c\hat{p}_k\hat{H}^{-1}. \tag{5}$$

Finally, the expressions of the velocity and position operators obtained are [4]:

$$\hat{v}_k(t)=c\hat{\alpha}_k(t) = \underbrace{c^2\hat{p}_k\hat{H}^{-1}}_{\hat{v}_k^{CM}} + \underbrace{c(\hat{\alpha}_k - c\hat{p}_k\hat{H}^{-1})e^{-\frac{2i\hat{H}t}{\hbar}}}_{\hat{v}_k^{ZBW}(t)}, \tag{6}$$

$$\hat{x}_k(t)=\underbrace{c^2\hat{p}_k\hat{H}^{-1}t}_{\hat{x}_k^{CM}(t)} + \underbrace{\frac{i}{2}\hbar c\hat{H}^{-1}(\hat{\alpha}_k - c\hat{p}_k\hat{H}^{-1})e^{-\frac{2i\hat{H}t}{\hbar}}}_{\hat{x}_k^{ZBW}(t)}. \tag{7}$$

The motion of the particle can be then decomposed into 2 components:

- The global motion of the center of mass (CM) of the particle, with average speed v^{CM} (relativistic group velocity of the particle);
- An oscillatory Zitterbewegung (ZBW) motion of high frequency and amplitude r.

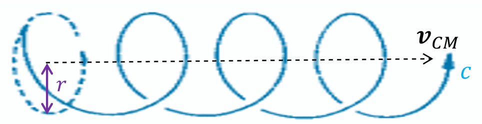

Figure 1: Illustration of Dirac particle average motion with total velocity c, longitudinal speed v_{CM} and zitterbewegung motion of amplitude r.

1.3 Analysis of Zitterbewegung equations

The expectation value of a quantum operator $\hat{O}$ is: $\langle\hat{O}\rangle = \langle\psi|\hat{O}|\psi\rangle$, with ψ the wavefunction of the particle considered. For example, the expectation value of the wave packet Hamiltonian is $\langle\psi|\hat{H}|\psi\rangle = E$. Moreover, we will use the normalization condition $\langle\psi|\psi\rangle = 1$.We have:

$$\langle\psi|\hat{v}_k^2|\psi\rangle = \langle\psi|c^2\hat{\alpha}_k^2|\psi\rangle = c^2\langle\psi|I|\psi\rangle = c^2. \tag{8}$$

So the eigenvalues of the mean velocity operator are $\pm c$, which means that the global velocity of the Dirac particle has a magnitude equal to the speed of light.

As expected, the average value of the center-of-mass component of the velocity operator gives the classical relativistic speed:

$$\langle\psi|\hat{v}_k^{CM}|\psi\rangle = \langle\psi|c^2\hat{p}_k\hat{H}^{-1}|\psi\rangle = \frac{c^2 p_k}{E} = v_k^{CM}. \tag{9}$$

Finally, the average value of the zitterbewegung component of the position operator is:

$$\langle\psi|(\hat{x}_k^{ZBW})^2|\psi\rangle = \left(\frac{\hbar c}{2E}\right)^2 \left(1 - \left(\frac{cp_k}{E}\right)^2\right).$$

At rest ($v_k^{CM} = 0/p_k = 0$), there is only the Zitterbewegung motion; it is a circular motion at the speed of light with average radius $r = \frac{\hbar c}{2E} = \frac{\hbar}{2mc}$.

2 Light-like internal dynamics of the electron at rest

"According to our present conceptions the elementary particles of matter are, in their essence, nothing else than condensations of the electromagnetic field" Einstein [11]

In agreement with the light-like oscillatory motion of the zitterbewegung, we propose the interpretation that an electron corresponds to a massless gauge field (like a photon) confined into a loop $\mathcal{C}$ with tangential speed c (speed of light). The internal periodic motion postulated is the "wrist-watch timer" of the fundamental particle to reckon the "proper time" in its rest frame. The particle can be compared to a small clock of period T (see Figure 2).

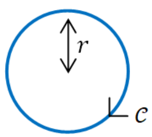

Figure 2: Illustration of the average circular trajectory of the massless gauge field, producing a light clock of radius r.

The energy of the particle is equal to that of a confined wave of frequency ν and wavelength λ: $E = h\nu = \frac{hc}{\lambda}$, with h the Planck constant. A photon curled into a stable resonating orbit manifests itself as a massive quantum particle of energy $E = mc^2 = h\nu$. De Broglie described this equivalence between the proper mass and the frequency of the oscillatory motion as "*a great law of Nature*".

2.1 Experimental facts

Some experimental effects are in favor of this view:

- Pair production/annihilation ($\gamma + \gamma \leftrightarrow e^- + e^+$) suggests that charged leptons and photons are different topological states of the same fundamental field. Pair production is seen here as a conversion process of electromagnetic energy from a linear path into a localized closed loop, perceptible as particles with mass.
- The Einstein-de Haas effect highlights the close relation between the spin angular moment of elementary particles and the kinetic moment of solids in rotation [12];
- Light rays scattering experiments led by Compton showed that the electron has a diameter comparable with the wavelength of γ rays [13]. He pointed out that a flexible ring electron accurately accounts for the difference between emergent and incident scattered radiation.

Why does the electron behave as point-like in high-energy collision experiments? What is measured in high-energy scattering experiments is the cross section and not the size of the particle; the cross section represents only a reflection zone within the electron and not its actual physical radius.

2.2 Quantum features of the electron at rest

In this part, we will study in detail the quantum features of the particle obtained from the Zitterbewegung motion in the rest frame.

2.2.1 Radius and frequency of the internal motion

As seen before, the mean radius of the zitterbewegung motion at rest is:

$$r = \frac{\hbar}{2mc} \approx 1.93 \times 10^{-13} \text{ m.} \tag{10}$$

The average frequency is:

$$\omega = \frac{2}{\hbar}\langle\psi|\hat{H}|\psi\rangle = \frac{2E}{\hbar} = \frac{2mc^2}{\hbar}. \tag{11}$$

One can notice that the radius is connected to this frequency by the formula $r = \frac{\hbar}{2mc} = \frac{c}{\omega}$, which clearly indicates a circular motion.

What is the relation between this radius and the wavelength of the confined wave?

$$\lambda = \frac{hc}{E} = \frac{h}{mc} = 4\pi r. \tag{12}$$

Fermions, like the electron, are described by Dirac spinor [14]. They are associated with a symmetry of 4π: two full rotations are needed to come back to the initial state, as shown in the previous formula.

2.2.2 Spin angular momentum

After the discovery of spin by Pauli, George Uhlenbeck and Samuel Goudsmit associated the new quantum number with the angular momentum corresponding to the rotational movement of the electron on itself [15]. This angular moment had a value of $\hbar/2$ and could only take two possible directions in the presence of an external magnetic field. This quantum number would then correspond to an additional degree of freedom of the electron, its intrinsic rotation. However, the calculations carried out by Pauli showed that in the case of an electron of finite size with a homogeneously distributed charge, the speed on the surface of the electron must be considerably higher than the speed of light, which contradicts relativity. Indeed, noting I the moment of inertia and ω the angular frequency, we have for a spherical distribution:

$$S = I\omega = \tfrac{2}{5} m r_e^2 \tfrac{v}{r_e} = \tfrac{2}{5} m v r_e.$$

$$\text{Moreover, } S = \tfrac{\hbar}{2} \rightarrow v = \tfrac{5\hbar}{4 m r_e} = \tfrac{5\hbar}{4 m \alpha \frac{\hbar}{mc}} = \tfrac{5c}{4\alpha} \approx 171c.$$

Physicists deduced that the electron is a point-like particle and does not have a finite size. Spin is considered by modern physics to have no equivalent in the classical world, it is purely quantum.

However, in our model, we don't have a spherical distribution of charge at $r_e \sim 10^{-15}$ m scale. The electron consists in a light-like loop motion at Compton scale $\sim 10^{-13}$ m. The angular momentum generated by this rotational motion is, by noting $\boldsymbol{p}_c$ the momentum of the confined photon:

$$S = |\boldsymbol{r} \times \boldsymbol{p}_c| = \frac{\hbar}{2mc}\, mc = \frac{\hbar}{2}. \tag{13}$$

which corresponds in quantum mechanics to the norm of the spin in a given direction.

The orientation of the spin (up or down) depends on the direction of rotation of the confined photon and on the observer. Spin can then be viewed as a real rotational motion, corresponding to a circulating flow of energy [16]. As Dirac said: *"The spin angular momentum of a particle should be pictured as due to some internal motion of the particle"* [17].

2.2.3 Origin of the mass and confinement

The energy of a photon confined in a given region of space (such as a box or a loop as in this particle model) contributes to an effective mass [18]:

$$m = \frac{\text{photon energy}}{\text{photon speed}^2} = \frac{E}{c^2}. \tag{14}$$

Moreover, a loop carrying energy and momentum manifests a resistance to acceleration, so this system has inertia.

However, to really understand the origin of the mass, we need to understand the origin of the confinement of a massless gauge field on a loop. What is the cause of this confinement? We propose that it is related to the Higgs mechanism. In particle physics, the Higgs field is responsible for generating the mass of elementary particles (like the electron). We note $v \approx 246$ GeV the vacuum expectation value of the Higgs field ϕ and g the coupling constant between the Higgs and a massless gauge field A_μ with symmetry $U(1)$. In the spontaneous symmetry breaking (SSB) mechanism, the Higgs boson combines to a gauge field A_μ; the coupling term between the Higgs and the gauge field can be expressed as [19]:

$$|D_\mu \phi|^2 = \left| \left(\partial_\mu + i\frac{g}{\sqrt{2}} A_\mu \right) \phi \right|^2 = \frac{1}{2} g^2 v^2 A^\mu A_\mu = \frac{1}{2} \left(\frac{mc}{\hbar} \right)^2 A^\mu A_\mu. \tag{15}$$

This mechanism produces a massive particle of mass $m = gv$. The Lagrangian obtained is $\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \left(\frac{mc}{\hbar} \right)^2 A^\mu A_\mu$; the field equation obtained is the Klein Gordon equation (wave equation for massive spin 0 particles):

$$\left(\partial_\nu \partial^\nu + \left(\frac{mc}{\hbar} \right)^2 \right) A_\mu = 0, \tag{16}$$

which makes appear the Compton radius $r_c = \frac{\hbar}{mc}$.

Moreover, the Higgs mechanism has a direct analogy with the Anderson-Higgs mechanism in superconductivity. In this SSB mechanism, a "free" photon interacts with a charged condensate (Cooper pairs), and it becomes short-range (confined on a limited spatial extent λ typically of the order of 100 nm). The electromagnetic gauge symmetry is broken, and the photon then acquires an effective mass because of its interaction with the charged condensate. The effective mass is:

$$m = \frac{\hbar}{\lambda c},$$

in direct analogy with the mass of a massive elemntary particle $m = \frac{\hbar}{r_c c}$. There is a good chance that the Higgs mechanism is responsible for the confinement of the gauge field A_μ, to form a massive particle like an electron. In this approach, in the case of fermions, the dynamical scalar Higgs boson can confine the electromagnetic flow of the photon into a non-trivial topology (spinor) with effective mass, half-integer spin and charge.

3 Dynamics of the particle and relativity

3.1 Helical motion

From the laboratory frame, the moving particle has a helical structure on average, in agreement with the Zitterbewegung motion. This helical motion is associated with non-null helicity H. The helicity is positive (or Right) when the spin points towards the same direction as motion; it is negative (or Left) when the spin points towards the opposite direction as motion (see Figure 3).

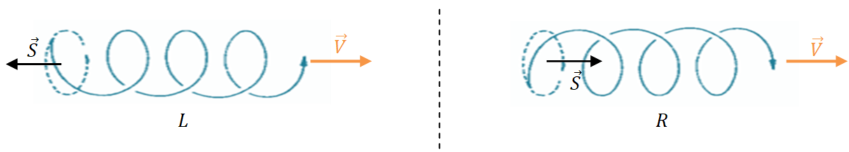

Figure 3: Illustration of helicity L/R

Moreover, we understand intuitively why the massive particle cannot go faster than c: as it has an internal dynamic at the speed of light, the longitudinal speed cannot exceed the speed of light: $V < c$.

3.2 General relativity and curved space-time

This quantum model of particles shares the same conceptions as Einstein's General Relativity (describing gravity and space-time): realism, causality and real trajectories. This circular motion at rest is associated with radius and curvature ($\kappa = \frac{1}{r} \sim \frac{mc}{\hbar}$). We see here that an important curvature is associated with a small radius and a high mass, which is consistent with General relativity. Einstein was never satisfied with the energy-momentum source of his gravitational field equation, because of its non-geometric character; the spin-zitter model corrects this deficiency, as it reduces mass to frequency in the curvature of light-like particle path. The curved world line of a massive particle corresponds to the trajectory of the axis of the helix (see Figure 4).

Figure 4: Curved trajectory in the case of a massive particle (with speed $V < c$)

In De Broglie view, the internal vibration of the particle creates waves in space, and in exchange this matter wave guides the motion of the particle. This can be put in correspondence with Einstein's view, where particles curve spacetime and in exchange the gravitational field determines the motion of particles along the geodesics of space. Those ideas of De Broglie and Einstein, expressed in different forms, are one of the greatest ideas in physics of the 20^{th} century. We can also mention that this type of particle model with real spin angular momentum naturally produces torsion in Riemann-Cartan spacetime [20]; it generalizes relativity theory and provides a more complete account of local gauge invariance with respect to the Poincaré group describing elementary particles.

Conclusion

The analysis of Zitterbewegung motion allows us to better understand what an electron is. First, it is a wave paquet, a superposition of plane waves; this wave paquet is localized. The zitterbewegung gives us information about the average trajectory of the wave paquet: it is a circular trajectory at the speed of light at rest, with radius $\frac{\hbar}{2mc}$ and frequency $\frac{2mc^2}{\hbar}$; and in motion, a helical trajectory at the speed of light and longitudinal speed $V < c$.

The light-like circular/helical structure of elementary particles explains naturally the origin of the spin (as real angular momentum), mass (energy confined on closed loop) and helicity L/R.

Such a particle can be represented as light-like vortex in space-time. The shape, size and motion of the particle are modified according to the interactions with other elementary particles and electromagnetic fields from its environment. These reactions to external field changes constitute the emission or absorption of energy for the particle. For example, if the confined gauge field absorbs energy, then the frequency and mass of the particle will increase. For the electron, the probability of emission or absorption of a photon is the fine structure constant α.

The internal quantum structure of the electron can be generalized to the other leptons μ and τ, they just differ by their radius/mass/frequency: $r_e > r_\mu > r_\tau \leftrightarrow m_e < m_\mu < m_\tau \leftrightarrow \nu_e < \nu_\mu < \nu_\tau$. The other types of massive elementary particles of the standard model can be generated from other confinement topologies, associated internal symmetries $U(1)$, $SU(2)$ or $SU(3)$. Topology defines group transformations, and the group transformation rules justify the algebra underlying the differential equations of motion.

Some other points remain to be investigated: origin of the three generations of leptons, topological description of other elementary particles like the proton, nature of the electric charge,... Those points will be developed in future work.

"You know, it would be sufficient to really understand the electron" Einstein

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(Manuscrit reçu le 15 décembre 2025)