Zero radiated power from unexcited atoms and atomic stability

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ABSTRACT. Larmor's well-known formula for the power radiated by accelerating electric charges does not provide correct results for atomic electrons in unexcited states (which do not radiate). No attempt at providing a complete solution to the phenomenon of non-emission from atoms (and therefore of atomic stability) has been generally accepted. An equation for radiated power is proposed which is applicable to atomic electrons in unexcited states. This equation is arrived at without adhering to a specific interpretation of quantum mechanics and is a step towards a more agreeable quantitative account of why atoms are stable.

RÉSUMÉ. La formule bien connue de Larmor pour la puissance rayonnée par charges électriques qui s'accélèrent ne fournit pas de résultats corrects pour les électrons atomiques dans des états non excités (qui ne rayonnent pas). Aucune tentative d'apporter une solution complète au phénomène de la non-émission des atomes (et donc de la stabilité atomique) n'a été généralement acceptée. Un équation pour la puissance rayonnée est proposée, qui est applicable aux électrons atomiques dans des états non excités. Cette équation est arrivé à sans adhérer à une interprétation spécifique de la mécanique quantique et c'est un pas vers une explication quantitative plus acceptable de la raison pour laquelle les atomes sont stables.

1 Introduction

The model of the atom proposed by Rutherford (circa 1911) has many well-known inadequacies. The most crucial of these is that an electron would spiral into an atom's nucleus in a time interval of between 10^{-8} and 10^{-11} seconds [1, 2]. The reason for this catastrophic motion being that the electron would lose energy by radiating electromagnetic waves. The prediction of these emissions comes from the classical electrodynamics result that accelerating electric charges radiate. Such emissions are overwhelmingly es-

tablished as they are the source of all television, radio and microwave transmissions. This (supposed) catastrophic motion of atomic electrons leads to the problem of how to account for the stability of atoms (as atoms clearly do not collapse).

If this result of classical electrodynamics did actually apply to an atom's electrons then the following would be the case. An electron in an unexcited atom would:

- constantly emit electromagnetic waves at a specific frequency if the electron moved around the nucleus at a fixed distance; or
- continuously emit electromagnetic waves of varying frequencies if the electron moved around the nucleus whilst also altering its distance from the nucleus.

However, neither (on-going) constant frequency nor continuously varying frequency emissions of electromagnetic waves from atoms have been detected — the only emissions found are of discrete frequencies for short, definite durations.

Bohr's model of the atom (circa 1913-15) avoids energy loss but only by stipulating that unexcited electrons have stable orbits around the nucleus in which they do not radiate [3, 4]. In other words, emissions and the accompanying energy loss from atoms in the Bohr model are avoided *by decree* thereby (theoretically) yielding stable atoms. This is hardly a satisfactory resolution. There have been several attempts made to provide physically sound explanations for the stability of atoms since 1915 but no single explanation has been found unanimously acceptable by physicists and physical chemists. Although there are many discussions available on the topics of radiation emission from atoms and atomic stability, their quality differs significantly. We shall begin by briefly summarizing the principal attempts at finding a solution to the conundrum of the stability of atoms. This will be followed by an outline of the problems with these attempts. A new equation for radiated power which is applicable to unexcited atomic electrons will then be presented and it will be contended that a re-examination of the problem of the stability of atoms should be undertaken.

2 Principal Attempts to Explain the Stability of Atoms

We shall now very briefly consider five post-1915 explanations for the stability of atoms. These constitute the principal endeavours to account for such stability. There are other explanations to be found in print and on the internet but these are not of a sufficient standard to warrant attention.

2.1 Orthodox Quantum Theory

The advent of Orthodox Quantum Theory in the 1920s, i.e. the widespread acceptance of the quantum mechanical formalism together with the Copenhagen interpretation, dealt with many difficulties arising from the Bohr model. The rationale offered by advocates of Orthodox Quantum Theory for the stability of atoms supplies (in their view) a sufficient explanation for atomic stability. This rationale has two parts.

The first part is the claim that there are no trajectories for quantum particles in Orthodox Quantum Theory until relevant measurements are performed [5]. It is then asserted that, without trajectories for atomic electrons, the classical electromagnetic behaviour of accelerating electrically charged particles is *not applicable*. Therefore, it is inferred that electromagnetic emissions do not occur from atoms except for when atomic electrons change their energy levels.

The second part employs the momentum-position uncertainty relation, as understood in Orthodox Quantum Theory. The momentum-position uncertainty relation is usually written as [6]:

$$\Delta x \, \Delta p \geq \hbar/2 \ldots (1)$$

where Δx is the uncertainty in a particle's position, Δp is its momentum uncertainty, and $\hbar$ is Planck's Constant divided by 2π. This is interpreted in Orthodox Quantum Theory as expressing a fundamental indeterminacy in the values of these two physical quantities. Since equation (1) requires that the product of the uncertainties in momentum and position has a non-zero lower bound, the individual uncertainties Δx and Δp cannot have zero values. When an electron approaches closer and closer to the nucleus, its position uncertainty becomes smaller, i.e. $\Delta x \to 0$ with $\Delta p \to \infty$ and its momentum becomes enormous so that the electron would be freed from the atom (which obviously does not occur) [7]. Worse still, if an electron were to move into the centre of the nucleus, then $\Delta x = 0$. Such motion is forbidden as it would violate the uncertainty relation as interpreted in Orthodox Quantum Theory. Equation (1) sets a minimum position uncertainty consistent with the electron being bound to the nucleus, so circumventing in Orthodox Quantum Theory the possibility of the electron 'falling' into the nucleus.

2.2 Applied Quantum Electrodynamics

In classical electrodynamics, if a specific region of volume V contains electrically charged particles with an amount of total charge q then the charge

density of this region is: $\rho = (q/V)$. If these charged particles have an average velocity **v** then the electric current density vector is: $\boldsymbol{j} = \rho \mathbf{v}$. In Orthodox Quantum Theory, the function $|\psi|^2$ (the probability density) assigns a probability that a given observable quantity will be found to have a particular value on measurement within set limits (e.g. in a spatial region) where ψ is the wavefunction of the system with $|\psi|^2 = \psi\psi^*$ (ψ^* being the complex conjugate of ψ). Quantum electrodynamics is a quantum field theory about the interactions of charged particles with a quantized electromagnetic field. Since quantum electrodynamics is a quantum theory, the formalism and interpretation of Orthodox Quantum Theory are taken to hold in quantum electrodynamics as well as specific field theoretical postulates.

It is claimed that quantum electrodynamics when applied to atomic electrons shows that these electrons do not emit electromagnetic waves. Charge density in quantum electrodynamics is defined in probabilistic terms to be: $\rho_c = q|\psi|^2$. The probability current density vector ($\boldsymbol{S}$) is the change per unit time of the probability of finding a particle in a given interval and is correlated with a system's wavefunction. It is quantitatively defined as [8]:

$$\boldsymbol{S} = \frac{\hbar}{2mi} [\psi^*(\nabla\psi) - \psi(\nabla\psi^*)] \ldots (2)$$

where m is the relevant inertial mass, ∇ is the standard differential vector operator, and i is the square-root of minus one. In atomic stationary states (i.e. the states of unexcited atomic electrons), an electron's probability density $|\psi|^2$ is time-independent. Since it can be shown that $\boldsymbol{j}$ is proportional to $\boldsymbol{S}$, current density is also time-independent [9]. The claim is then made that no electromagnetic radiation is emitted in atomic stationary states as Maxwell's equations of classical electrodynamics require a varying current density in order to generate electromagnetic waves.

2.3 de Broglie-Bohm Quantum Theory

One of the primary competitors to Orthodox Quantum Theory is the de Broglie-Bohm quantum theory (although it is viewed by many physicists as another interpretation of the quantum formalism since its statistical predictions are identical to those of Orthodox Quantum Theory). In de Broglie-Bohm theory, a quantum particle is always accompanied by a (physical) quantum field. Quantum particles have positions and velocities (and consequently trajectories) at all times, not just when being measured. Similar to

other physical fields, the quantum field (described mathematically by the wavefunction) has a potential function (Q) which is given by [10]:

$$Q = -\frac{\hbar^2}{2m}\left(\frac{\nabla^2 R}{R}\right) \quad \ldots (3)$$

where $R = |\psi|$ is the magnitude of the quantum particle's wavefunction and m is the particle's inertial mass. The quantum field exerts a force on the quantum particle, called the quantum mechanical force, which affects the particle's motion. This force is a pure quantum effect and does not appear in the equations of classical mechanics [11]. The quantum mechanical force ($\boldsymbol{f}$) is found from equation (3) by taking the vector gradient of the potential function Q:

$$\boldsymbol{f} = -\nabla Q = \frac{\hbar^2}{2m}\nabla\left(\frac{\nabla^2 R}{R}\right) \quad \ldots (4)$$

Every atomic electron is bound by the Coulomb force originating from the nucleus, i.e. $-Ze/(4\pi\varepsilon_o r^2)$, where Ze is the charge of the nucleus with e being the electron's charge, Z is the number of protons in the nucleus, r is the electron-nucleus distance, and ε_o is the electric permittivity constant. In the de Broglie-Bohm account of the atom, all electrons are subject to both the Coulomb force and the quantum mechanical force such that these forces counteract each other preventing electrons 'falling' into the nucleus, i.e.:

$$|\boldsymbol{f}| \geq Ze/(4\pi\varepsilon_o r^2) \quad \ldots (5)$$

Exact calculations using de Broglie-Bohm equations show that electrons in bound atomic states do move in definite trajectories around the nucleus without 'falling' in [12]. However, as every atomic electron must be accelerating on its trajectory (for even a change in direction produces an acceleration), there would still be constantly and/or continuously radiated electromagnetic waves.

The question now is — why have these waves not been detected? Unfortunately, de Broglie-Bohm theory does not explain this non-detection.

2.4 Confinement Energy

An explanation for the stability of atoms has also been provided in terms of an electron's kinetic energy. This explanation has been popular with physical chemists [13–16]. Consider the total energy of the single electron in a

Hydrogen atom. This total energy E is the sum of the electron's kinetic energy (K) and its electric potential energy (V):

$$E = K + V = \frac{e^2}{8\pi\varepsilon_o r} + \left(\frac{-e^2}{4\pi\varepsilon_o r}\right) = -\frac{e^2}{8\pi\varepsilon_o r} \quad \ldots (6)$$

where the symbols are as previously defined. The potential and total energies are negative as the electron is bound to the atom's nucleus. It can be seen from equation (6) that both an electron's kinetic energy (also called 'Confinement Energy') and its potential energy will continue to increase in their respective magnitudes as the electron gets closer to the nucleus. Losses of energy from the electron are assumed to occur by electromagnetic emissions.

The formalism of quantum mechanics requires that the energy of a bound particle is quantized and has a minimum value in its ground state. A balance between kinetic and potential energies is postulated to occur in the ground state so that there are no more changes in energy and the total energy becomes constant. Once the ground state has been reached (a state of minimum energy), electromagnetic emissions cease [17]. This prohibition on electromagnetic emissions is taken for granted and *not* further expounded upon. Atomic stability is (in part) explained in this account by an energy balance and the quantization of energy which, together, is taken to prevent electrons from spiralling into the nucleus.

The same question arises — if atomic electrons do constantly and/or continuously radiate electromagnetic waves (at least for part of the time of their motion about the nucleus), why has this not been detected?

2.5 Zero-Point Energy of Space

A different line of attack to the conundrum of the stability of atoms assumes the existence of the (so-called) zero-point energy of space. Stochastic electrodynamics is a classical theory about charged particles which exist in a radiation filled space. This theory postulates that empty physical space (the 'vacuum') contains energy which is not macroscopically accessible [18, 19] (although its presence is inferred by measurements of the Casimir Effect at low temperatures, i.e. the tiny force between two very close metal plates). Stochastic electrodynamics posits that physical space has a minimum energy which is not zero (the zero-point energy).

In the stochastic electrodynamics explanation of atomic stability, an electron does lose energy by radiating electromagnetic waves but at the lowest level of energy, the electron absorbs an amount of energy supplied by the physical space surrounding it which is equal to the energy lost by electro-

magnetic emissions. Consequently, at this lowest level, the rates of energy changes involving an individual atomic electron are related by the equality:

$$-\frac{dE_{\text{rad}}}{dt} = \frac{dE_{\text{abs}}}{dt} \quad \ldots (7)$$

where E_{rad} is the energy radiated by the electron and E_{abs} is energy absorbed from the 'vacuum'. Equation (7) indicates that these emission and absorption processes take place in a state of dynamic equilibrium in which the atom is stable.

Numerical simulations for nonrelativistic electrons using stochastic electrodynamics show that this equality of energy loss and energy gain holds for closed paths in the neighbourhood of Bohr orbits without the electron 'falling' into the nucleus [20]. Other calculations have indicated that an atomic electron will absorb too much energy and be ejected from the atom [21], which obviously does not occur with stable atoms. Advocates of stochastic electrodynamics (a relatively small number of researchers) continue to seek solutions within this approach (e.g. see [22, 23]). Yet, here again, we have the unanswered question of why constantly and/or continuously radiated electromagnetic waves from atoms have not been detected.

3 Problems with the Principal Explanations

The Orthodox Quantum Theory rationale for the stability of atoms has the following difficulties. The claim that atomic electrons do not have trajectories if unmeasured which makes classical electromagnetism not applicable at atomic scales is little more than a conjecture. This is asserted in spite of classical electromagnetic theory working for other atomic scale phenomena, such as the generation of radio frequency waves from oscillating electrons inside an antenna. This conjecture is an interpretative aspect of Orthodox Quantum Theory as the quantum formalism by itself does not require any ban on particle trajectories [24, 25]. Indeed, some interpretations of quantum mechanics (including de Broglie-Bohm theory) use particle paths in their descriptions of quantum phenomena (e.g. see: [26–30]).

In addition, the meaning of the momentum-position uncertainty relation is not the same in all interpretations of quantum mechanics [31], e.g. the relation is also understood in terms of a disturbance on the state of a quantum particle induced by the measurement process. The Orthodox Quantum Theory rationale, therefore, cannot be drawn directly from the quantum formalism but instead, is only inferred from interpretative aspects.

In respect to the application of quantum electrodynamics to atomic electrons, it is the case that quantum electrodynamics does provide correct pre-

dictions for atomic phenomena involving charged particles, e.g. photon absorptions and emissions. However, it does not give an explanation of why unexcited atoms do not radiate that is independent of interpretation as the meaning of the function $|\psi|^2$ varies in different interpretations of quantum mechanics (see also [32]). Moreover, if the application of quantum electrodynamics did provide an explanation which was acceptable to all physicists and physical chemists, there would not be any other contemporary attempts to explain atomic stability. The Orthodox Quantum Theory rationale and/or applied quantum electrodynamics cannot be used if another interpretation is accepted which does not share relevant attributes.

Now it is the case that any electron revolving about an atom's nucleus at a fixed distance should constantly radiate electromagnetic waves of a frequency equal to the frequency of revolution of the emitting electron. If the electron was at a distance between the first and second Bohr radii say, then the electromagnetic waves that would be constantly emitted would lie in the ultraviolet region of the spectrum. These waves should not differ greatly in frequency or intensity from those electromagnetic waves emitted when an electron 'falls' from the second to the first energy level, i.e. as found in discrete emission line spectra [33]. If electromagnetic waves were constantly and/or continuously emitted then they would have to be of an intensity below the threshold of detection. This is highly unlikely in view of the (measurable) intensity of the electromagnetic waves emitted by electrons when 'falling' to lower energy levels.

Since constantly nor continuously emitted electromagnetic waves from unexcited atoms have not been found and, in the absence of any coherent explanation for extremely low intensity of these waves, the conclusion which follows is that atomic electrons do not so radiate. Therefore, it is *not* the case that atomic electrons radiate electromagnetic waves and that the lost energy is restored to them as the waves are emitted. If atomic electrons do not constantly nor continuously radiate then any explanation of atomic stability in which such emissions are present can be excluded. This includes de Broglie-Bohm, Confinement Energy and Zero-Point Energy explanations as they currently stand. An explanation of atomic stability should be sought which is not dependent on any particular interpretation of quantum mechanics *and* in which constantly/continuously radiated electromagnetic waves from atoms are not emitted.

4 Larmor's Formula for Radiated Power and its Shortcomings

Classical electromagnetic theory, as it stood in the early 20th century, predicted atomic electrons would emit electromagnetic waves with the power (P) radiated by a non-relativistic accelerating charged (point) particle given by Larmor's Formula [34]:

$$P = \frac{q^2}{6\pi\varepsilon_o c^3} a^2 \quad \ldots (8)$$

where $a = a(t)$ is the magnitude of the (instantaneous) acceleration vector ($\boldsymbol{a}$) of the particle at time t, q is the amount of charge and c is the speed of light in vacuum. Equation (8) can *only* yield a zero value for power P if $a = 0$, but zero acceleration is not possible for an electron in motion about an atom's nucleus (recall that any change in direction produces an acceleration). Further, although the relativistic version of equation (8) shows greater power is radiated when a particle's speed is close to the speed of light, it does not assist in explaining the absence of atomic emissions.

Equation (8) is experimentally confirmed for unbound charged particles accelerating through empty space [35]. Given that atomic electrons do accelerate, Larmor's formula predicts that unexcited atoms should radiate. However, the finding that electromagnetic waves are not emitted by unexcited atoms shows that equation (8) cannot be correct for atomic electrons.

5 An Equation for Radiated Power Applicable to Atomic Electrons

Whatever the correct explanation is for the non-emission of electromagnetic waves from unexcited atomic electrons, it is clear that equation (8) fails for them. Consequently, a new equation for radiated power is needed for unexcited atomic electrons (which gives a zero value) and does not depend on any interpretation of quantum mechanics. This equation would have the following properties:

(i) more than one term in the power expression; and
(ii) valid only for bound, unexcited electrons in atoms.

Property (i) is necessary so that the power radiated can be zero even when acceleration is not zero. Property (ii) includes the conditions which pertain to the motion of atomic electrons and is necessary as equation (8) is confirmed for unbound, accelerating charged particles.

A radiated power equation applicable to atomic electrons in unexcited states can be arrived at without adhering to a single interpretation of quantum mechanics if the starting point is Larmor's formula and suitable extra terms are added to it with the requirement that the resulting equation yields a relation which describes the motion of atomic electrons. This equation

would apply not just to the ground state of an atom but to all unexcited states. It would not apply to excited states nor to atomic transitions as these necessitate the use of quantum mechanical techniques. Let's emphasise again the empirical motivation for seeking this equation — unbound, accelerating electrons do radiate and with a power given by Larmor's formula whereas atomic electrons in unexcited states do not radiate at all.

It is legitimate to employ the mathematical apparatus of particle paths as no particular interpretation of quantum mechanics is being imposed. The motion of a bound electron about the nucleus will be described here by a general orbital model. It is important to note that this description is not incompatible with quantum trajectory methods, e.g. see the treatment in Holland (1994) [36]. What is more, this can be done without any need for a Fourier decomposition. We shall only consider a single electron to avoid the complications of inter-electron interactions. Since an electron bound to an atom's nucleus moves under the influence of a central, inverse square force (i.e. the Coulomb force), its motion lies in a plane [37] and is described by three basic variables.

We choose standard plane polar coordinates (r, θ) for this purpose (as this affords the most straightforward description) which are functions of the third variable, time t. Then the radial vector ($\boldsymbol{r}$), which extends (and points) from the nucleus to the electron, has the form:

$$\boldsymbol{r} = r\,\hat{\mathrm{e}}_r \quad \ldots (9)$$

where $r = r(t)$ is the electron-nucleus distance at time t and $\hat{\mathrm{e}}_r$ is a (non-constant) unit vector in the direction of the radial vector. The electron's velocity vector ($\mathbf{v}$) and acceleration vector ($\boldsymbol{a}$) are found by taking time derivatives of equation (9):

$$\mathbf{v} = \dot{r}\,\hat{\mathrm{e}}_r + r\dot{\theta}\,\hat{\mathrm{e}}_\theta \quad \ldots (10)$$

and

$$\boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\mathrm{e}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\mathrm{e}}_\theta \quad \ldots (11)$$

where the dots indicate differentiation with respect to time and $\hat{\mathrm{e}}_\theta$ is a (non-constant) unit vector at right angles to $\hat{\mathrm{e}}_r$.

However, because the force on the electron ($\boldsymbol{F}$) is always directed towards the nucleus, its acceleration vector may be found by applying Coulomb's Law:

$$\boldsymbol{a} = (\boldsymbol{F}/m) = (-k/mr^2)\,\hat{\mathrm{e}}_r \quad \ldots (12)$$

where the constant $k = Ze^2/(4\pi\varepsilon_o)$, m is the electron's inertial mass, and other symbols are as previously defined. Equations (11) & (12) allow the magnitude of the electron's acceleration to be expressed as:

$$a = (-k/mr^2) = (\ddot{r} - r\dot{\theta}^2) \quad \ldots (13)$$

Multiplying equation (13) by r^3 and rearranging gives:

$$r^3\ddot{r} - (r^2\dot{\theta})^2 + (k/m)\,r = 0 \quad \ldots (14)$$

In a central force field, angular momentum is conserved, so that the quantity $w = (r^2\dot{\theta})$ is a constant of motion. Equation (14) then becomes

$$r^3\ddot{r} + (k/m)\,r - w^2 = 0 \quad \ldots (15)$$

Now consider the extra terms required so that the electron's radiated power can be zero even though its acceleration is non-zero. These terms need to be scalars and have units of acceleration squared for consistency with equation (8). There are two (non-redundant) terms which fit these conditions and are expressible using the variables of the general orbital model. These terms are the following scalar products:

$$\mathbf{v} \cdot (d\boldsymbol{a}/dt) \quad \ldots (16)$$

and

$$\boldsymbol{r} \cdot (d^2\boldsymbol{a}/dt^2) \quad \ldots (17)$$

An equation for radiated power which applies to unexcited atomic electrons (i.e. for which $P = 0$ even though $a \neq 0$) may be found if we take $q = -e$ in equation (8) and add terms (16) & (17) such that:

$$P = \left(\frac{e^2}{6\pi\varepsilon_o c^3}\right)\left[a^2 + c_1\left(\boldsymbol{r}\cdot\frac{d^2\boldsymbol{a}}{dt^2}\right) + c_2\left(\mathbf{v}\cdot\frac{d\boldsymbol{a}}{dt}\right)\right] \quad \ldots (18)$$

where c_1 and c_2 are (non-zero) real-valued dimensionless constants, to be determined.

In order to find the constants c_1 and c_2, we obtain expressions for terms (16) & (17) by taking time derivatives of equation (12) and forming the relevant scalar products using equations (9) & (10). The resulting terms are expressed as functions of radial distance and its time derivatives:

$$\mathbf{v} \cdot \frac{d\boldsymbol{a}}{dt} = \frac{k}{mr^5} \; (2r^2\dot{r}^2 - \mathrm{w}^2) \; \cdots (19)$$

and

$$\boldsymbol{r} \cdot \frac{d^2\boldsymbol{a}}{dt^2} = \frac{k}{mr^5} \; (2r^3\ddot{r} - 6r^2\dot{r}^2 + \mathrm{w}^2) \; \cdots (20)$$

If we insert the expressions from equations (19) & (20) into equation (18) then for power $P = 0$, the quantity inside the square brackets in equation (18) becomes:

$$\frac{k^2}{m^2r^4} + \left(\frac{k}{mr^5}\right) \left\{\mathrm{c}_1(2r^3\ddot{r} - 6r^2\dot{r}^2 + \mathrm{w}^2) + \mathrm{c}_2(2r^2\dot{r}^2 - \mathrm{w}^2)\right\} = 0 \; \cdots (21)$$

where $a^2 = (k^2/m^2r^4)$ from equation (12). It can be seen that equation (21) will hold for an attractive, central inverse square force field if $\mathrm{c}_1 = 1/2$ and $\mathrm{c}_2 = 3/2$, as substituting these values into equation (21) and rearranging yields:

$$r^3\ddot{r} + (k/m)\, r - \mathrm{w}^2 = 0 \; \cdots (22)$$

which is identical to equation (15), i.e. the equation describing the motion of atomic electrons. Hence, we have arrived at the desired equation for radiated power for unexcited atomic electrons:

$$P = \left(\frac{e^2}{6\pi\varepsilon_o c^3}\right)\left[a^2 + \frac{1}{2}\left(\boldsymbol{r} \cdot \frac{d^2\boldsymbol{a}}{dt^2}\right) + \frac{3}{2}\left(\mathbf{v} \cdot \frac{d\boldsymbol{a}}{dt}\right)\right] \cdots (23)$$

The result of attaining equation (22) by way of equations (18), (19) and (20) shows this to be so.

If an unexcited electron were to cease being held by an atom's nucleus then equation (23) would no longer apply to this electron. This is straightforward in the case of an electron ejected from an atom as the electron's motion beyond the atom's electric field will be linear with a constant velocity so that the terms $(d\boldsymbol{a}/dt)$ and $(d^2\boldsymbol{a}/dt^2)$ are both zero. Equation (23) would then automatically reduce to Larmor's formula.

Equation (23) being classical in its form is a consequence of beginning with Larmor's formula and employing the general orbital model. This suffices to find the extra terms needed to give zero radiated power for atomic electrons in unexcited states. In view of the fact that the non-emission from unexcited states is electromagnetic in nature, it is reasonable to expect that

equation (23) will (eventually) be derived from first principles by applying classical electromagnetic theory to the circumstances pertaining to the state of an unexcited atomic electron (cf. Section 7). Such a derivation would also provide the physical meaning of the two extra terms in equation (23). Incidentally, Feynman, Morinigo and Wagner published an equation containing a term similar to the last one in equation (23) but they did not find an expression for zero radiated power for unexcited atomic electrons [38].

6 Two Illustrative Examples

Equation (23) will now be used in two examples to demonstrate that it will hold for an unexcited electron bound to an atom's nucleus.

Example One. Consider first the ground state of the Hydrogen atom. This may be represented by an electron having a circular orbit about the nucleus of a fixed radius r and a constant speed v. Then the electron's acceleration ($\boldsymbol{a}$) is just its centripetal acceleration given by:

$$\boldsymbol{a} = -(\mathrm{v}^2/r)\hat{\mathrm{e}}_r \quad \ldots (24)$$

Note that equation (24) was not utilized in obtaining equation (23). The time derivatives of acceleration in equation (23) are non-zero for *all* motion about a nucleus including when the magnitude of the radial vector is fixed. Using equations (9), (10) & (24), we have:

$$\boldsymbol{r}\cdot(d^2\boldsymbol{a}/dt^2) = (\mathrm{v}^2/r)^2 \text{ and } \mathbf{v}\cdot(d\boldsymbol{a}/dt) = -(\mathrm{v}^2/r)^2 \quad \ldots (25)$$

If we substitute equations (25) into equation (23), we find the value of the power radiated from the single electron in the ground state of Hydrogen:

$$P = \left(\frac{e^2}{6\pi\varepsilon_o c^3}\right)\left[\left(\frac{\mathrm{v}^2}{r}\right)^2 + \frac{1}{2}\left(\frac{\mathrm{v}^2}{r}\right)^2 - \frac{3}{2}\left(\frac{\mathrm{v}^2}{r}\right)^2\right] = 0 \quad \ldots (26)$$

which is the expected zero result.

Example Two. The paths of particles bound by an attractive, central inverse square force field (e.g. the Coulomb field) are, in general, elliptical. The radius vector $\boldsymbol{r}(\theta)$ for an object on an elliptical orbit is [39]:

$$\boldsymbol{r}(\theta) = \mathrm{b}\left[(1-\varepsilon^2)/(1-\varepsilon\cos\theta)\right]\hat{\mathrm{e}}_r = (\beta/\alpha)\,\hat{\mathrm{e}}_r \quad \ldots (27)$$

where b (> 0) is the size of the semi-major axis of the elliptical orbit, ε is its eccentricity with $0 \le \varepsilon < 1$, $\beta = \mathrm{b}(1-\varepsilon^2)$ and $\alpha = (1-\varepsilon\cos\theta)$. Applying equations (10) & (11) together with equation (27) shows that:

$$\boldsymbol{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\mathrm{e}}_r = [(-\varepsilon w/\beta)\dot{\theta}\cos\theta + (-\beta\dot{\theta}^2/\alpha)]\,\hat{\mathrm{e}}_r \quad \cdots (28)$$

Differentiating equations (27) & (28) with respect to time, forming the relevant scalar products and substituting into the square brackets in equation (23) gives:

$$\left(\frac{\varepsilon w}{\beta}\right)\left\{\left(\frac{\varepsilon w}{\beta}\right)\dot{\theta}^2\cos^2\theta + \frac{1}{2}r\dot{\theta}^3\cos\theta - \left(\frac{\varepsilon w}{\beta}\right)^2\left(\frac{3\dot{\theta}}{r}\right)\sin^2\theta\cos\theta\right.$$
$$- \left(\frac{\varepsilon w}{\beta}\right)\dot{\theta}^2\cos^2\theta + 3\left(\frac{\varepsilon w}{\beta}\right)\dot{\theta}^2\sin^2\theta - \frac{3}{2}\left(\frac{\varepsilon w}{\beta}\right)\dot{\theta}^2\sin^2\theta$$
$$\left. - \frac{1}{2}r\dot{\theta}^3\cos\theta + \left(\frac{\varepsilon w}{\beta}\right)^2\left(\frac{3\dot{\theta}}{r}\right)\sin^2\theta\cos\theta - \frac{3}{2}\left(\frac{\varepsilon w}{\beta}\right)\dot{\theta}^2\sin^2\theta\right\} = 0 \quad \cdots (29)$$

which again is the expected zero result.

7 Prospects for Further Development

Progress made in classical electromagnetism over the last few decades (e.g. see: [40–43]) indicates that reconsidering the behaviour of charged particles in motion under different conditions will lead to new insights. Deviations from equation (8) will arise, for example, when a charged particle's electric field is altered in the presence of an external electric field [44]. This opens up the possibility that the explanation of why unexcited atoms do not emit electromagnetic waves (and consequently are stable) can still be informed by classical electromagnetism. Other aspects of atoms, i.e. excited states and atomic transitions would, of course, require quantum mechanical methods for their descriptions and computations.

The recent advances in classical electromagnetic theory and the presentation of a new equation applicable to atomic electrons in unexcited states (equation (23)), suggests that an explanation of the stability of atoms acceptable to both physics and chemistry communities is achievable. In particular, equation (23) should be derivable from first principles by the application of classical electromagnetism to bound, unexcited atomic electron states. An atomic electron is an extended physical entity (not a point) and is always moving within the electric field of an atom's nucleus. Consequently, an atomic electron's electric field is modified by interaction with the field of the nucleus. It then follows that the derivation of equation (23) would need to take into account the finite size of the electron and the distortion to its electric field due to the nuclear electric field. This derivation would constitute a major step towards a more acceptable (quantitative) explanation of

atomic stability which is independent of any interpretation of quantum mechanics.

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